Chapter 8
Trig Equations and Applications

Lesson Plans and Class Notes

Snell's Law

$$\frac{\text{speed of light in air}}{\text{speed of light in water}} = \frac{\sin \alpha}{\sin \beta}$$

8.1 Simple Trigonometric Equations

Lesson Objective:

- solve and apply simple trigonometric equations
- find the inclination of a line

****Reference angles**** they don't know when to add pi or sub pi or subtract from 2pi

sin x = 0.5 has two solutions between 0 and 360 degrees.

30 and 150

Notice that the 150 degree angle has the same sine ratio because it makes a 30 degree angle with the x-axis. 30 degrees is the reference angle.

The equation "sin x = 0.5" has many solutions.

The unit circle tells us the $\pi/6$ and $5\pi/6$ are two *particular solutions*.

Since any coterminal angle from the above solutions will also be a solution, we describe the *general solution* as:

$$x = {}^{\pi}/_{6} + 2n\pi \text{ or } x = {}^{5\pi}/_{6} + 2n\pi$$

We can use a similar method to find the general solution for a ratio not found on the unit circle:

$$\sin x = 0.6$$

In our calculator, we can use Sin^{-1} to find the angle between $-{}^{\pi}/_{2}$ and ${}^{\pi}/_{2}$.

$$\mathrm{Sin}^{-1}\, 0.6 \approx 0.6435$$

But there is another *particular solution* that occurs between ${}^{\pi}/_{2}$ and π . (This is the other quadrant where sine is positive.)

$$\mathrm{Sin}\, 2.4981 \approx 0.6$$

(Note: $\pi - 0.6435 = 2.4981$)

Therefore:

The general solutions to the equation

"$\sin x = 0.6$"

$$x \approx 0.6435 + 2n\pi \text{ or } x \approx 2.4981 + 2n\pi$$

Turn to your neighbor and discuss:

The solutions to sin x = 0.5 were:

$x = {}^{\pi}/_{6} + 2n\pi$ or $x = {}^{5\pi}/_{6} + 2n\pi$

The solutions to sin = 0.6 were:

$x \approx 0.6435 + 2n\pi$ or $x \approx 2.4981 + 2n\pi$

#1 - - - Why did the first problem use "equal signs" and the second problem need "approximately equal to" signs?

#2 - - -Why do the solutions include the term "$2n\pi$"?

You can solve the equation:

$$3x + 9 = 7$$

(You would subtract 9, and then divide by 3.)

Now replace x with a trigonometric function:

$$3 \cos \theta + 9 = 7$$

Follow the same steps to isolate the trigonometric function:

$$\cos \theta = {}^{-2}/_{3}$$

Now solve using Cos^{-1} and find the general solutions.

$$\Theta \approx 131.8^{\circ} + 360n \text{ or } \Theta \approx 228.2^{\circ} + 360n$$

Finding inclination of a line:

Inclination of a line is measurement of the positive angle formed by the line and the x-axis. Inclination will always be an angle between 0 and 180 degrees.

$$m = \tan \alpha$$

Proof of this theorem is on page 297.

Find the inclination of the line $2x + 5y = 15$

1. Find the slope by rewriting in slope-intercept form.
2. Plug slope into the Inclination formula
3. Solve for the angle

$$y = {}^{-2}/_{5}x + 3$$

$${}^{-2}/_{5} = \tan \alpha$$

$$\alpha = \mathrm{Tan}^{-1}\ {}^{-2}/_{5} \approx -21.8^{o}$$

Note: α must be an positive angle. The only positive angles that have negative tangent ratios are in the second quadrant.

Therefore, $\alpha \approx 180 – 21.8 \approx 158.2^{o}$

Snell's Law

(Refer to page 300 #45)

You spot a fish while spear-fishing and estimate that your line of sight makes a 25 degree angle with the still water.

(This creates a 65 degree angle “to the normal.”)

At what angle to the normal do you need to throw your spear to get this fish?

As you get closer to the fish, will the angle your line of sight makes “to the normal” increase or decrease?

Will the angle of incidence β increase of decrease?

Will the fish ever be exactly where we think it is?

8.2 Sine and Cosine Curves

Lesson Objectives:

- alter the graphs of sine and cosine to apply to electricity problems

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The graph
$y = \sin x$
has a period of 2π and
an amplitude of 1.

Consider the graph of
$y = A \sin x$

How do different values of A change the graph?
If A is positive....
If A is negative...
If $0<A<1$...
If $A = 1$...

Consider the graph

$y = \sin Bx$

How do different values of B change the graph?

If B is positive....

If B is negative...

If $0<B<1$...

If $B = 1$...

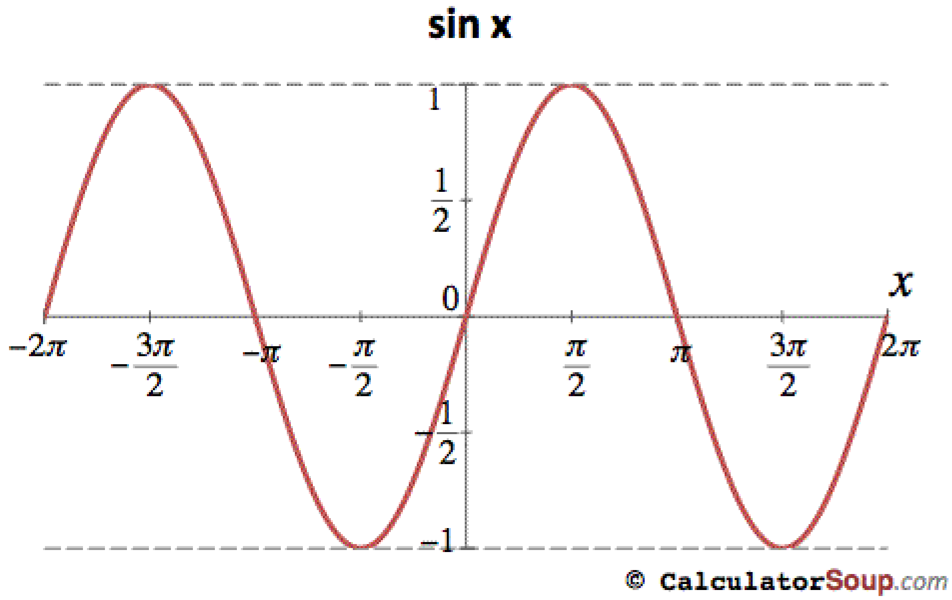
Sine and Cosine Curves

For functions:

$y = A \sin Bx$ and $y = A \cos Bx$

amplitude $= |A|$ and period $= {}^{2\pi}/_{B}$

Sketch a graph of y = -4 sin 3x



Think:
This graph has a bigger amplitude.
It is flipped over the x-axis.
The period is shorter.

Write the equation of the given graph:

- How do you know to use sin or cos?
- How do you find the amplitude?
- How do you find the period?

Applications to Electricity

Read “Applications to Electricity” pg 303 – 304.

Then answer:

What is the voltage equation for an AC circuit that oscillates at a frequency of 40 cycles per second and delivers energy at the same rate as a direct current of 150 volts?

Homework: pg 305 1-17odd

8.3 Modeling Periodic Behavior

Lesson Objective:

- translate graphs of sine and cosine horizontally and vertically to model periodic behavior

Investigate:

In the last section, we investigated how A and B change the graphs of sine and cosine in the following equations:

$$y = A \sin Bx \qquad \text{or} \qquad y = A \cos Bx$$

Now we will investigate how h and k change the graphs:

How do values for "h" alter the graphs of sine and cosine?

$$y = \sin (x - h) \qquad \text{or} \qquad y = \cos (x - h)$$

(Shift the cosine curve left pi/2 units. What do you notice?)

How do values for "k" alter the graphs of sine and cosine?

$$y = \sin x + k \qquad \text{or} \qquad y = \cos x + k$$

General Sine Waves

In your notes, describe how A, B, h, and k alter the graph of sine:

$$y = A \sin [B(x - h)] + k$$

__To find the equation of a sine wave graph:__

1. Find the amplitude by dividing the difference of the maximum and minimum value by 2

2. Use the period equation to find the period of the graph.

3. Find the axis wave by averaging the max and min value.

4. Find the horizontal translation.
   a) If cosine, select the highest point and determine the distance from the y-axis.
   b) If sine, select an x-intercept and determine the distance from the y-axis.

The depth of water at the end of a pier varies with the tides throughout the day. Today, the high tide occurs at 4:15am with a depth of 5.2m. The low tide occurs at 10:27am with a depth of 2m.

a) sketch a graph of the tide as a function of time vs. depth

b) find the depth of the water at noon

c) If a large boat needs at least 3m of water to moor at the end of the pier, during what times can the boat moor?

Homework: pg 313 #1-7 odds, 15, 17

8.4 Identities and Equations
Relationships Among the Functions

Lesson Objectives:

- simplify trigonometric expressions
- prove trigonometric identities

Reciprocal Relationships

Relationships with Negatives

Pythagorean Relationships

Cofunction Relationships

Use page 317 and 318 as reference.

Note: $\sin^2 x = (\sin x)^2$

Examples:

Simplify	**Video**
$\tan^3 x \bullet \csc^3 x$	http://youtu.be/SZu_EVV4jjY
$\sec x \bullet \cos x - \cos^2 x$ $(\csc^2 x - 1)(\sec^2 x \sin x)$	http://youtu.be/FS6iQX7jY-s
$\dfrac{\csc^2 x - 1}{\csc^2 x}$ $\dfrac{\csc^2 x - \cot^2 x}{\tan^2 x - \sec^2 x}$	http://youtu.be/FS6iQX7jY-s

To prove identities:

- Prove one side is equal to the other side.
- Sometimes, it is easy to show both sides simplify to the same expression.

8.5 Solving Complex Equations

Lesson Objectives:

- Solve complex trigonometric functions

Hints:

- Rewrite so that you only have one kind of trigonometric function.
- Isolate the trigonometric function.
- If you cannot isolate it, try factoring.